1. Starting from time zero

- Lifetime is random variable $T > 0$
- Survival function, $S(t) = \Pr(T > t)$
- Clearly, $0 \leq S(t) \leq 1$ and $S(t)$ is a non-increasing function of t
- Distribution function, $F(t) = \Pr(T \leq t)$
- Probability density function, a.k.a. *curve of deaths*:

$$f(t) = \lim_{h \to 0^+} \frac{1}{h} \Pr(t \leq T < t + h)$$

- Hazard function, a.k.a. *force of mortality*:

$$\mu_t = \lim_{h \to 0^+} \frac{1}{h} \Pr(t \leq T < t + h | T \geq t)$$

- Integrated hazard function, $H(t) = \int_0^t \mu_s ds$.

2. Some relationships

- $F(t) = 1 - S(t)$
- $f(t) = S(t)\mu_t$
- $S(t) = \exp\left(-\int_0^t \mu_s ds\right)$
- $S(t_1 + t_2) = S(t_1)S(t_2)$ for $t_1, t_2 > 0$

3. Now assume life is already aged x

- *Future* lifetime is random variable $T_x > 0$
- *Actual total* lifetime is $x + T_x$
- Survival function, $S_x(t) = \Pr(T_x > t)$
- Distribution function, $F_x(t) = \Pr(T_x \leq t)$
- Probability density function, a.k.a. *curve of deaths*:

$$f_x(t) = \lim_{h \to 0^+} \frac{1}{h} \Pr(t \leq T_x < t + h)$$

- Hazard function, a.k.a. *force of mortality*:

$$\mu_{x+t} = \lim_{h \to 0^+} \frac{1}{h} \Pr(t \leq T_x < t + h | T_x \geq t)$$

4. Some more relationships

- $S_x(t) = \frac{S(x+t)}{S(x)}$

5. World of actuaries

- $S_x(t)$ is called ${}_tp_x$
- $F_x(t)$ is called ${}_tq_x$
- ${}_1p_x$ is shortened to p_x
- ${}_1q_x$ is shortened to q_x
- ${}_tq_x = \int_0^t {}_sp_x \mu_{x+s} ds$
- ${}_tp_x = \exp\left(-\int_0^t \mu_{x+s} ds\right)$
- ${}_hq_x \approx h\mu_x$ for small h.
- Explore these relationships at http://www.richardsconsulting.co.uk/mortality_interface.html