Lecture cancelled at 11:15hrs on Friday 10^{th} **February 2006**

Mortality expectations

- Above age 25, we expect mortality to increase steadily with age
- From age 60 onwards we expect mortality to increase exponentially with age

  ⇒ Or, equivalently, linearly on a log scale.
  ⇒ $\log q_x = \alpha + \beta x$, i.e. Gompertz' Law (see graph MortalityLaws.pdf).
- At all ages we expect males to have heavier mortality than females

  ⇒ See http://www.the-actuary.org.uk/pdfs/04_02_09.pdf for why.
- We expect population mortality to be heavier than insured mortality
- At shorter durations we expect lighter mortality than at longer durations

  ⇒ known as *temporary initial selection*
- We expect direct-sales business to have heavier mortality than intermediated business
- We expect amounts-weighted mortality to be lighter than lives-based mortality

  ⇒ Policy size (asset share, pension payment, sum insured etc) is often used as a crude proxy for socio-economic group. Since wealthier people have lower mortality rates than poorer ones, and since wealthier people also have larger policies, amounts-weighted mortality measures invariably produce lower answers than lives-based ones.

Some phenomena

- Late-life mortality deceleration

  ⇒ The rate of increase in mortality rates slows down at advanced ages
- Mortality convergence

  ⇒ Tendency for mortality differentials to reduce with age (except gender).
  ⇒ Contrast this with Cox model, which assumes **constant** proportion.

Perk's Law

- Simplified Perks' Law has $q_x = \dfrac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$.

  ⇒ Also called a *logistic curve* in mathematics.
  ⇒ $\text{logit } q_x = \log\left(\dfrac{q_x}{1-q_x}\right) = \alpha + \beta x$
- Above is a transformation of q_x to make it linear in age.
- Logit function is the *canonical link* for binomial data in a GLM, showing direct relationship between GLMs and an actuarial mortality law.
- $\dfrac{q_x}{1-q_x}$ is called the *odds ratio*, and is a natural statistic for proportions.
- $\log\left(\dfrac{q_x}{1-q_x}\right)$ is called the *log-odds ratio*.

Graduation

- Actuaries need to smooth observed rates before use

  ⇒ Need to remove effect of random variation.
  ⇒ Need to have regard to business purpose

- Balance goodness of fit versus smoothness

  ⇒ These are often in tension, and graduation or model-fitting seeks to balance the two.

Testing smoothness

- Third differences between observed and fitted values

  ⇒ Should be small relative to fitted values.
  ⇒ Should progress regularly.

- Note that fitting a model gives automatic smoothness, e.g. Gompertz Law

Testing goodness of fit

- Overall fit: the χ^2 test
- Importance of residuals

  ⇒ Are there any patterns in the residuals?
  ⇒ Are there too many large residuals in general?
  ⇒ Are there any unusually large residuals?