Tutorial 1
Solutions to tutorials are generally available at Iain Currie's homepage at:

http://www.ma.hw.ac.uk/~iain/teaching/mortality/mortality.html

However, the solution to question three (="prove $f_x(t) = S_x(t)\mu_{x+t}$") is perhaps not quite obvious. An alternative is given below.

Proof of $f_x(t) = S_x(t)\mu_{x+t}$
Start with the basic definition of the density function, $f_x(t)$, of the future lifetime, T_x:

$$f_x(t) = \lim_{h\to 0^+} \frac{1}{h} \Pr(t \le T_x < t+h)$$

and the basic definition of the hazard function, μ_{x+t}:

$$\mu_{x+t} = \lim_{h\to 0^+} \frac{1}{h} \Pr(t \le T_x < t+h | T_x > t)$$

Now recall the basic result for the conditional probability for two events A and B, where $A \subset B$:

$$\Pr(A|B) = \frac{\Pr(A)}{\Pr(B)}$$

which gives us $\Pr(A) = \Pr(B). \Pr(A|B)$ (remembering of course that this holds true if and only if $A \subset B$). We note that the time interval $[t, t+h) \subset [t, \infty)$ for any positive value of h, so we can apply this conditional probability argument to our survival probabilities:

$$\Pr(t \le T_x < t+h) = \Pr(T_x > t). \Pr(t \le T_x < t+h | T_x > t), \forall h > 0$$

This gives us the identity with which to complete our proof:

$$\begin{aligned} f_x(t) &= \lim_{h\to 0^+} \frac{1}{h} \Pr(t \le T_x < t+h) \\ &= \lim_{h\to 0^+} \frac{1}{h} \Pr(T_x > t). \Pr(t \le T_x < t+h | T_x > t) \\ &= \Pr(T_x > t). \lim_{h\to 0^+} \frac{1}{h} \Pr(t \le T_x < t+h | T_x > t) \\ &= S_x(t).\mu_{x+t} \end{aligned}$$

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